An Elementary Illustration of Theoretical Understanding

Boyle's Law of Pressures and Volumes

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Introduction

The purpose of this paper is to provide an elementary illustration of first, an understanding of Boyle's Law, and secondly, aspects of our thinking process towards an understanding of Boyle's law.[1]

Boyle's law ($P \times V = C$) is presented in three ways. Historically, the first two presentations are in "reverse chronological order." But that reversal can prove helpful. The first is a pedagogical presentation of a "mock experiment." Results there are "too perfect." But the presentation can help you get a sense of the problem and also bring out the need for knowing some elementary mathematics.

Mock Experiment A

A platform is attached to the top of a plunger that fits into a rigid cylindrical tube. The plunger is designed both to create an air-seal and to move easily. Putting weights (in grams) onto the platform pushes the plunger to different levels. The more weights added, the further down the plunger goes. It is also observed that by removing weights, the plunger rises back to its original height. The trapped air seems to work like a spring. Reviewing some school mathematics, volume of a circular cylinder is (height) × (area of circular cross-section). In this mock experiment, the cross-sectional area of the cylinder is approximately one square cm (formula for the area of a circle). And so, to measure volume of trapped air, we only need to measure height of the plunger. A ruler is lined up alongside the cylinder.

[1] "Aspect of our thinking process" relates to Bernard Lonergan's approach called "generalized empirical method." See footnote 7, *Journeyism 5*, "How Does Intellectual Labour Proceed? (Part 3), https://bentonfuturology.com/journeyism5/ and *Journeyism 25*, "An Elementary Illustration of Theoretical Understanding, Galileo's Law of Falling Bodies," https://bentonfuturology.com/journeyism25/.

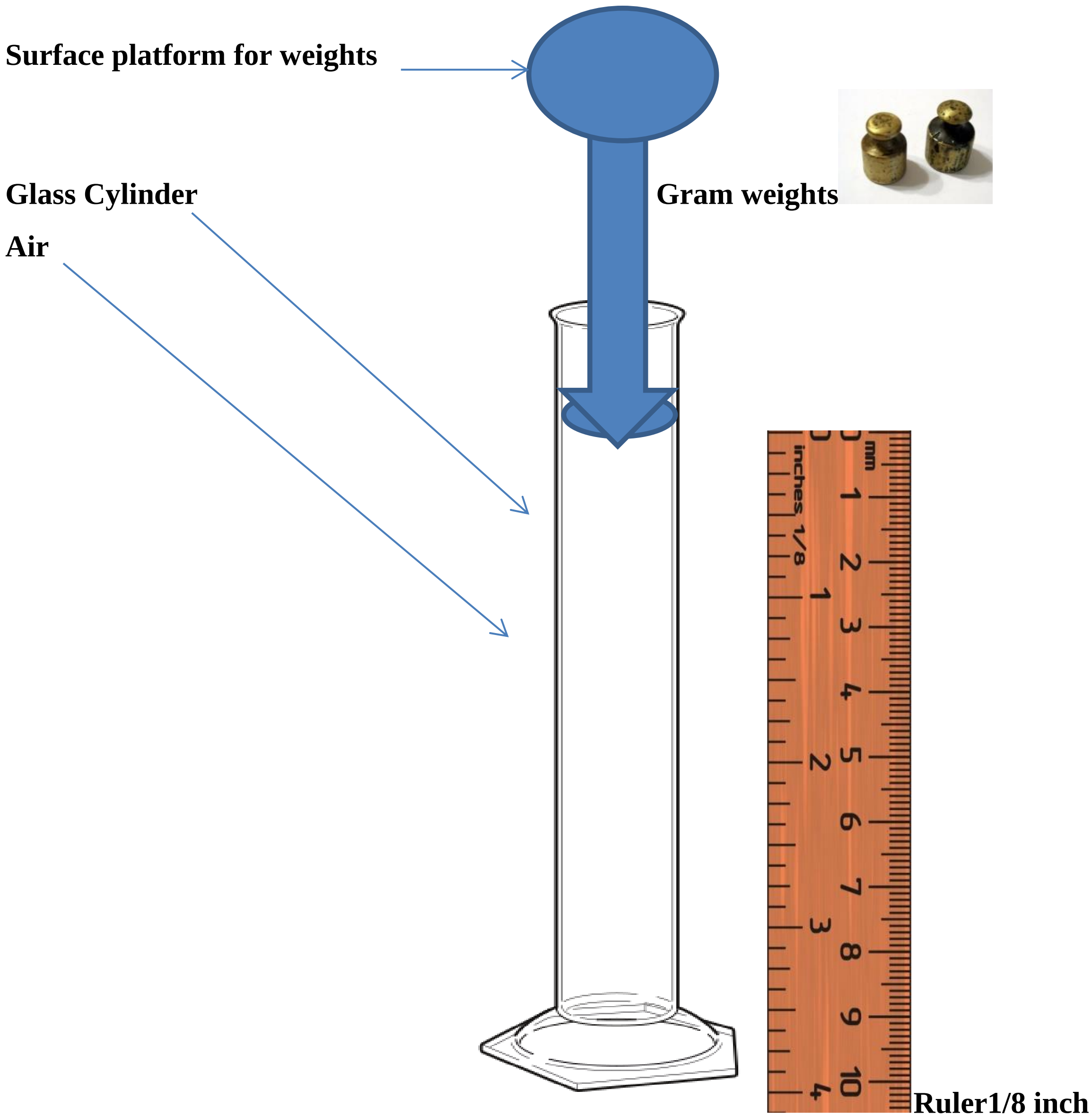

Figure 1. Mock Experiment A

Having results presented in a table can be helpful.

Weight on plunger (grams) for "pressure" P	Height of air under plunger (cm) for volume V
1	8
2	4
3	8/3
4	2
5	8/5
6	8/6
7	8/7
8	1

Table 1. Results from mock experiment A.

Notice, the larger the weight the smaller the volume. Can we be more specific? Another insight is called for. Do you have it? In each case, $P \times V = 8$. To tie in with the idea that smaller volumes result in larger pressures, you might find it helpful to write this as $V = 8/P$. (Divide both sides by P.)

Boyle's J-tube Experiment[2]

Boyle's experiment involved a "J-tube," a curved glass cylinder closed at one end.

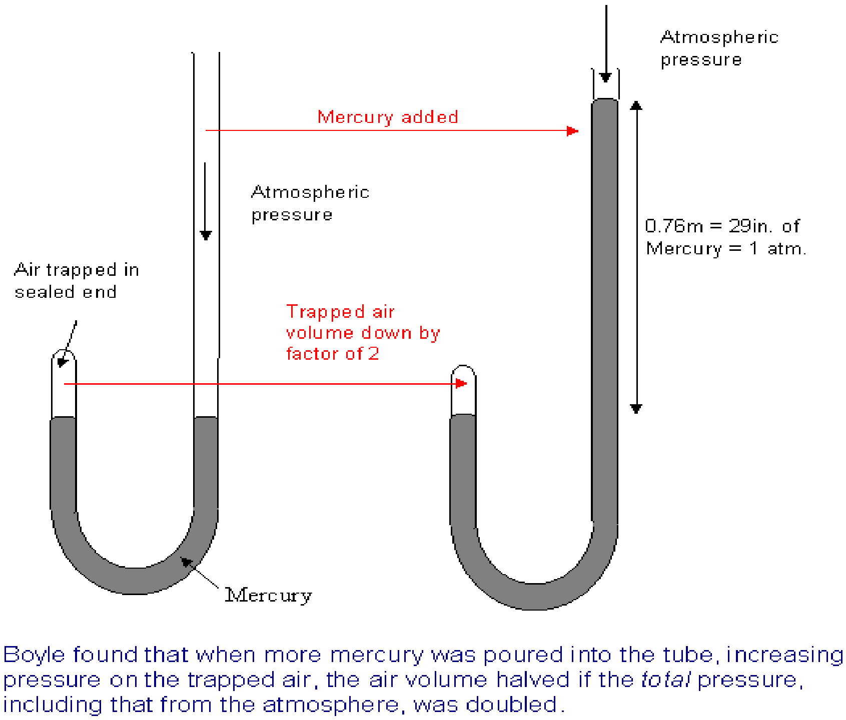

Figure 2. Boyle's J-tube experiment.

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[2] Similar images are easily found online. This one was obtained from, Michael Fowler, *Physics 152: Gravity, Fluids, Waves, Heat* (http://galileo.phys.virginia.edu/classes/152.mf1i.spring02/), "Boyle's Law and the Law of Atmospheres," http://galileo.phys.virginia.edu/classes/152.mf1i.spring02/Boyle.htm.

Pour “volumes of liquid mercury” into the open end of a “J-tube,” so-called because of its shape. Pressure is applied to a pocket of trapped air at the closed end. (See Figure 2.) Pour in a small amount of liquid mercury enough so that the mercury settles—as it happens, at the same level on both sides (as indicated in the left part of the diagram). On the hypothesis that added pressure (whatever pressure is) is proportional to the volume of mercury added, then to measure ‘pressure,’ we measure the height of added liquid mercury. We also measure the volume of trapped air in the same way. It is a clever set up. The pocket of air is much shorter than lengths along the column of mercury. So, the height of the air pocket is measured in quarter-inch steps, while liquid mercury is measured in full-inch steps. Here are some results that Boyle obtained in one of his experiments[3]:

Volume of trapped air (in quarter inches, experimental)	Pressure (volume of liquid mercury) in inches, experimental)	Product: P(experimental) V(experimental)
48	29 $^{2}/_{16}$	1398
46	30 $^{9}/_{16}$	1406
44	31 $^{15}/_{16}$	1405
42	33 $^{8}/_{16}$	1407
40	35 $^{5}/_{16}$	1413

Table 2. Partial results from Boyle’s experiment.

To refine his measurements, Boyle estimated 1/16th-inch increments. Looking across each row, the products of pairs of experimentally obtained P and V are not all the same (as they were in Mock Experiment A). But they are all close to being equal, to 1405, more or less. Again, a further insight is needed. Allowing for “experimental error,” P × V is (approximately) constant.

Boyle used many differently-sized J-tubes. With tools available to him at that time, constructing tubes that worked proved to be challenging. Sometimes columns burst under pressures from large volumes of liquid mercury. Persevering, what he found was that, by allowing for experimental error, P × V = constant, where the constant depended on the dimensions of the J-tube.

Mock Experiment B

This presentation is intended to help understand Boyle’s law. (See Figure 3.) It includes two (Cartesian) coordinate graphs for comparing pressures and volumes.[4] The first compares P and V directly. The second is for comparing 1/P with volume V. A graduated cylinder[5] is sealed with a plunger that can be pushed to various depths. A gauge measures pressures of trapped air.

Each pair of measurements (one for pressure and one for volume) provides coordinates on a “coordinate graph.” By joining dots on the graph (**not obtained experimentally**), we get the dark

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[3] John West, “The original presentation of Boyle’s Law,” *Journal of Applied Physiology*, vol. 87, no. 4 (1999), 1544, https://journals.physiology.org/doi/full/10.1152/jappl.1999.87.4.1543.
[4] Boyle was apparently aware of Descartes’ mathematical work but did not make use of coordinate graphs.
[5] The graduated cylinder was invented by Albert Einstein in 1909, in order to help with measuring the volume of a liquid.

curve in the picture. For large values of V, there are small values of P, and vice versa. More precisely, for each pair we get P × V = 195.

Taking a slightly different approach, if Boyle's law is correct, then (1/P) = (constant)V; that is, (1/P) is proportional to V. That means, in order to demonstrate Boyle's law, we could instead graph pairs 1/P and V. For, if pairs 1/P and V are in fact proportional, we should get a straight line, as seen in the second graph. Either way, we can see that the "mock experiment" "demonstrates" Boyle's law.

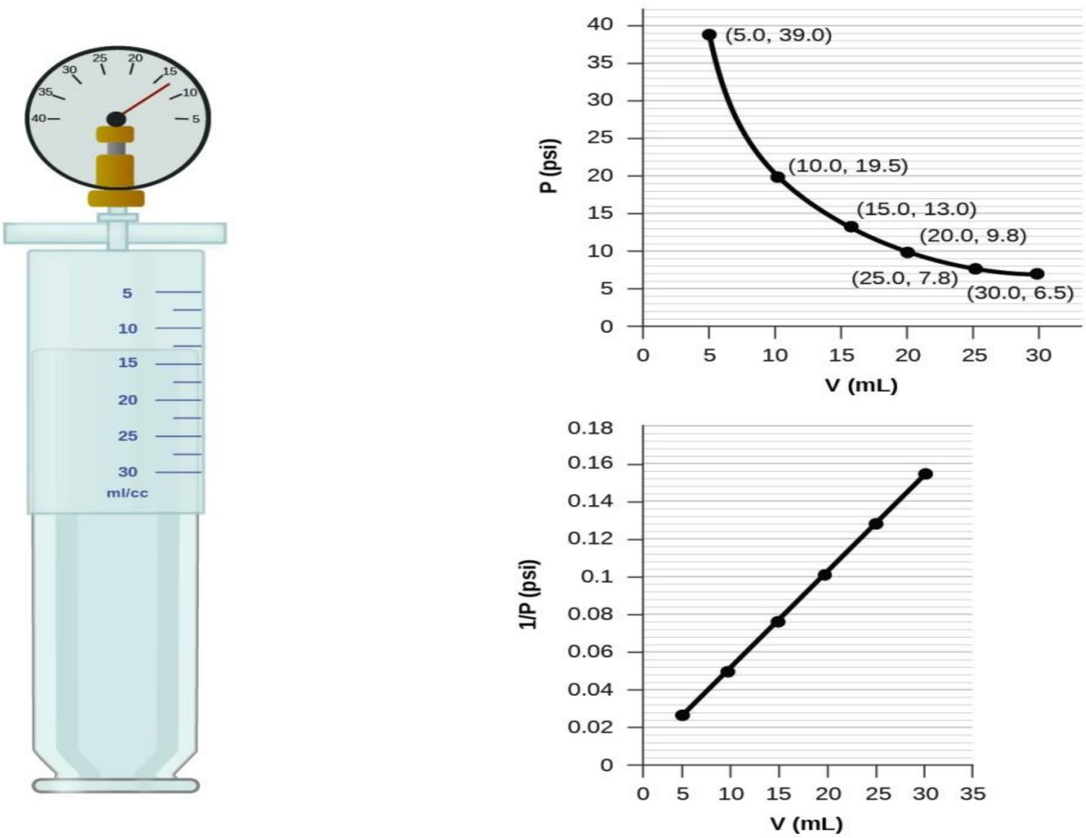

Figure 3. Mock Experiment B: A Demonstration of Boyle's Law

Discussion

We have provided an elementary illustration of first, an understanding of Boyle's Law, and secondly, aspects of our thinking process towards an understanding of Boyle's law.

The approach taken in Mock Experiment A and the J-tube experiment, respectively, demonstrate that understanding Boyle's law is the result of curiosity and insight. To generate each, they describe and then measure two sets of lengths using a standard unit of length. They gather numerical results in tables and graphs. By a further insight (which includes ignoring "experimental error"), they discover the possible relationship, P × V = Constant. Understanding goes beyond description[6] to grasp a relation between pairs of measured lengths that provisionally represent pressures and volumes.

This approach gives rise to further insights and understanding. For instance, scientific explanation calls for series of "correlations of correlations of correlations."[7] The challenge is to identify "compound" correlations in your own experience with further "self-attention."[8]

By contrast, Mock Experiment B is out of sync with this approach. Rather than evoke curiosity to promote insight into Boyle's law, Mock Experiment B begins by stating Boyle's law and directs students through an "arithmetic demonstration," to "what follows" from its application. This approach is an example of what I have routinely observed in contemporary education: a tendency to emphasize rote work (and other "nominal understanding"), rather than inquiry toward "explanatory understanding."

Boyle's law is not an isolated result but is used, verified, and demonstrated in conjunction with other understandings and technologies. For instance, historical developments led to "pressure due to weight" being better understood through Newton's laws of motion which, in turn, are used to design modern pressure gauges such as one might use for a bicycle tire.

Our exercise in Boyle's Law is a modest beginning to encourage attention to a future achievement of the scientific community called "generalized empirical method." In addition to discovering that P × V = Constant, you can also begin to advert to and notice aspects of your inquiry and understanding. It is, I think, important and in keeping with the invitation given by Bernard Lonergan, in his book *Insight*:

> In the midst of that vast and profound stirring of human minds, which we name the Renaissance, Descartes was convinced that too many people felt it beneath them to direct their efforts to apparently trifling problems.[9]

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[6] See *Journeyism 24*. Furthermore, understanding can easily be dodged by conceptual analysis and nominalism, the practice of which merely repeats the formula while covering up the absence of insight.
[7] For an explicit discussion of the triple correlation, in the context of 'things,' see p. 271 in Bernard Lonergan, *Insight: A Study in Human Understanding*, Collected Works of Bernard Lonergan, University of Toronto Press, 1992, (CWL3). See p. 113 for advanced heuristics, and p. 103 for description in Ch. 3, "The Canons of Empirical Method".
[8] Terrance Quinn, *Generalized Empirical Method: In Philosophy and Science*, (Hoboken, NJ: World Scientific, 2017), 21-26. Quinn discusses the triplicity of correlations in the context of Galileo's experiment on the law of falling bodies.
[9] Lonergan, *Insight* (1992), 27.

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